Economics 10A

Probability and Statistics in Economics I

Lecture 8
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http://www.researchmethods.org/10a

Moving Up from Descriptive

from 2 Es to 4 Es

Samples
n = 2
12 23 34 45
13 24 35
14 25
15

If sample size = 4
sample means range
from 2.5 to 3.5

Mean 2.5, 2.75, 3, 3.25, 3.5
Sample 1234 1235 1245 1345 2345

Population:
N=5
1 2 3 4 5

Mean = 3
Variance = 2

Sample Means
1.5 2.5 3.5 4.5
2 3 4
2.5 3.5
3

Mean of Samples Means = 3

Sample Variance
0.5 0.5 0.5 0.5
2 2 2
4.5 4.5
8

Mean of Samples Variances = 2.5

Easy confused?

- Say you have four shoes in your closet – a black pair and a red pair. You reach in without looking and pull out a shoe with each hand. What are the chances that you'll have a matching pair? One student says the probability is 2/3, because there are three probable outcomes: 1) both shoes are black, 2) both shoes are red, or 3) the shoes are different colors. Another student says the probability is only 1/2 , because there are four probable outcomes: 1) both shoes are black, 2) both shoes are red, 3) the shoe in your left hand is black and the shoe in your right hand is red, 4) the shoe in your left hand is red and the shoe in your right hand is black. Who is right?

What is probability?

Chance for some thing to happen?

- Probability is a numerical measure of the likelihood that an event will occur.
- Probability values are always assigned on a scale from 0 to 1.
- A probability near 0 indicates an event is very unlikely to occur.
- A probability near 1 indicates an event is almost certain to occur.
- A probability of 0.5 indicates the occurrence of the event is just as likely as it is unlikely.

P(E) = 0.48 - the probability for E to happen is 0.48

Experiment (process) and Sample Space

- An experiment is any process that generates well-defined outcomes.
- The sample space for an experiment is the set of all experimental outcomes.
- A sample point is an element of the sample space, any one particular experimental outcome.

Assigning Probabilities

Start from sample space sum of all probabilities of any sample space is always 1

- Classical Method
  - Assigning probabilities based on the assumption of equally likely outcomes.
- Relative Frequency Method
  - Assigning probabilities based on experimentation or historical data.
- Subjective Method
  - Assigning probabilities based on the assignor's judgment.

Example: Bradley Investments

Bradley has invested in two stocks, Markley Oil and Collins Mining. Bradley has determined that the possible outcomes of these investments three months from now are as follows.

Investment Gain or Loss in 3 Months (in $000)

Markley Oil	Collins Mining
10	8
5	-2
0	
-20	

A Counting Rule for Multiple-Step Experiments

- If an experiment consists of a sequence of k steps in which there are n_1 possible results for the first step, n_2 possible results for the second step, and so on, then the total number of experimental outcomes is given by $(n_1)(n_2) \ldots (n_k)$.
- A helpful graphical representation of a multiple-step experiment is a tree diagram.

Example: Bradley Investments

A Counting Rule for Multiple-Step Experiments

Bradley Investments can be viewed as a two-step experiment; it involves two stocks, each with a set of experimental outcomes.

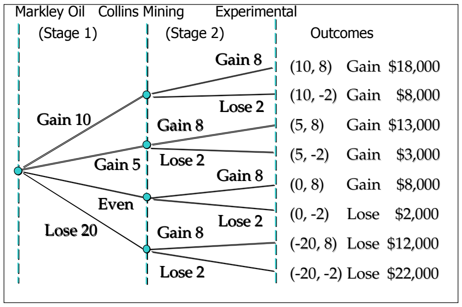
Markley Oil: $n_1 = 4$

Collins Mining: $n_2 = 2$

Total Number of Experimental Outcomes: $n_1 n_2 = (4)(2) = 8$

Example: Bradley Investments

- Tree Diagram



Counting Rule for Combinations

Another useful counting rule enables us to count the number of experimental outcomes when n objects are to be selected from a set of N objects.

- Number of combinations of N objects taken n at a time

$$C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!}$$

$N! = N(N-1)(N-2)\ldots(2)(1)$
$n! = n(n-1)(n-2)\ldots(2)(1)$
$0! = 1$

Counting Rule for Permutations

A third useful counting rule enables us to count the number of experimental outcomes when n objects are to be selected from a set of N objects where the order of selection is important.

- Number of permutations of N objects taken n at a time

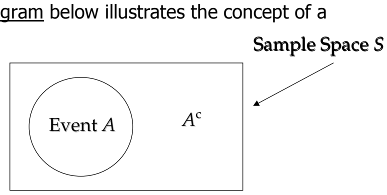
$$P_n^N = n!\binom{N}{n} = \frac{N!}{(N-n)!}$$

Some Basic Relationships of Probability

- There are some <u>basic probability relationships</u> that can be used to compute the probability of an event without knowledge of al the sample point probabilities.
  - Complement of an Event
  - Union of Two Events
  - Intersection of Two Events
  - Mutually Exclusive Events

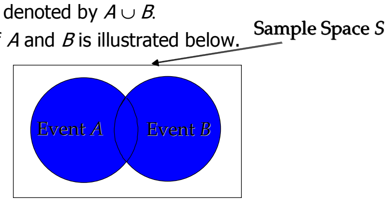
Complement of an Event

- The <u>complement</u> of event *A* is defined to be the event consisting of all sample points that are not in *A.*
- The complement of *A* is denoted by A^c.
- The <u>Venn diagram</u> below illustrates the concept of a complement.



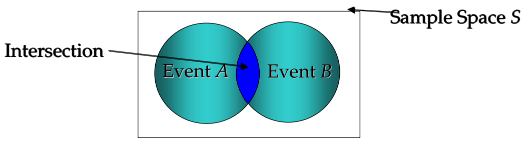
Union of Two Events

- The <u>union</u> of events *A* and *B* is the event containing all sample points that are in *A* or *B* or both.
- The union is denoted by $A \cup B$.
- The union of *A* and *B* is illustrated below.



Intersection of Two Events

- The intersection of events *A* and *B* is the set of all sample points that are in both *A* and *B*.
- The intersection is denoted by $A \cap B$.
- The intersection of *A* and *B* is the area of overlap in the illustration below.



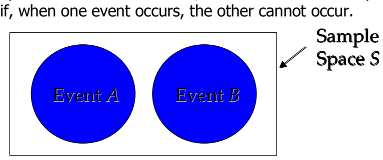
Addition Law

- The addition law provides a way to compute the probability of event *A,* or *B,* or both *A* and *B* occurring.
- The law is written as:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Mutually Exclusive Events

- Two events are said to be mutually exclusive if the events have no sample points in common. That is, two events are mutually exclusive if, when one event occurs, the other cannot occur.



- Addition Law for Mutually Exclusive Events

$$P(A \cup B) = P(A) + P(B)$$