

Combine Indices to Form a Single Measure

Dr. Alex Liu

Director of the RM Institute

Pasadena, CA 91101

Alex@ResearchMethods.org

Dec. 2006

A Common Problem in Social Science

- Concepts like:
 - Quality of environment
 - Effectiveness of training
 - Democracy
 - Governance
 - Loan Risk
 - Consumer power
 -
- are all a summary of
a collection of observable
quantities.

A Latent Variable Framework

- $f(m|I) = f(I|m) f(m) / f(I)$ – per Bayes' theorem

$I \sim I_1, I_2, I_3 \dots$ In observable.

$m \sim$ latent variable

$$f(I) = \int f(m) f(I|m) dm$$

$f(I|m)$ and $f(m)$ need to be determined
to obtain $f(m|I)$.

Scaling Methods

- If conditional independence and sufficiency condition hold, a formal statistical approach to be described later
- If not, many options exist:
 - summated rating scaling
 - is the most frequently used one.

Conditional Independence

- $f(I|m) = \prod f_i(I_i|m)$
-
- implies (1) only one factor
- (2) no interdependence among I s, except that through m .

Sufficiency Condition

- $f(\mathbf{I}|\mathbf{m}) = \prod f_i(I_i|\mathbf{m})$
- can be expressed by $f(\mathbf{\Pi}|\mathbf{m})$
- $\mathbf{\Pi} \sim$ a combination of I s
- then $f(\mathbf{m}|\mathbf{I}) \sim f(\mathbf{m}) f(\mathbf{\Pi}|\mathbf{m})$

Barankin & Maitra Theorem (1963)

- if conditional independence holds
- if
- $f(I_i|m) = G_i(I_i) H_i(m) \exp\{u_i(I_i) \Phi_i(m)\}$
- then
- $\Pi = \sum u_i(I_i)$
- then $f(m|I) \sim f(m) f(\sum u_i(I_i) |m)$

Special Cases

- I binary
- $u_i(I_i) = a_i * I_i$

- I Normal
- $u_i(I_i) = \lambda_i I_i / \sigma_i^2$

- u – a sufficient statistic and a base for forming a measure.

Our Work

- In the past, our group has built measures for social impacts of investment, training effectiveness, consumer purchasing, financial risk, entrepreneurship and democracy.

Our Findings

- We found that using $u_i(I_i) = \lambda_i I_i / \sigma_i^2$
- often performs better than that of using summated rating, even conditional independent and sufficient conditions do not hold.
- In general, a model based scaling method is always better than a simple addition approach based on intuition.

References

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