Combine Indices to Form a Single Measure

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A Common Problem in Social Science

- Concepts like:
- Quality of environment
- Effectiveness of training
- Democracy
- Governance
- Loan Risk
- Consumer power
- ……

are all a summary of
a collection of observable
quantities.

A Latent Variable Framework

- $f(m|I) = f(I|m)\, f(m) / f(I)$ – per Bayes' theorem

I ~ I1, I2, I3 … In observable.
m ~ latent variable
$f(I) = \int f(m)\, f(I|m)\, dm$

f(I|m) and f(m) need to be determined to obtain f(m|I).

Scaling Methods

- If conditional independence and sufficiency condition hold, a formal statistical approach to be described later
- If not, many options exist:

- summated rating scaling
- is the most frequently used one.

Conditional Independence

- $f(I|m) = \Pi\, f_i(I_i|m)$
-
- implies (1) only one factor
- (2) no interdependence among Is, except that through m.

Sufficiency Condition

- $f(I|m) = \Pi f_i(I_i|m)$
- can be expressed by $f(II|m)$
- II ~ a combination of Is
- then $f(m|I) \sim f(m) f(II|m)$

Barankin & Maitra Theorem (1963)

- if conditional independence holds
- if
- $f(I_i|m) = G_i(I_i)\, H_i(m) \exp\{u_i(I_i)\, \Phi_i(m)\}$
- then
- $II = \Sigma\, u_i(I_i)$

- then $f(m|I) \sim f(m)\, f(\, \Sigma\, u_i(I_i)\, |m)$

Special Cases

- I binary
- $u_i(I_i) = a_i * I_i$

- I Normal
- $u_i(I_i) = \lambda_i I_i / \sigma^2_i$

- u – a sufficient statistic and a base for forming a measure.

Our Work

- In the past, our group has built measures for social impacts of investment, training effectiveness, consumer purchasing, financial risk, entrepreneurship and democracy.

Our Findings

- We found that using $u_i(I_i) = \lambda_i I_i / \sigma^2_i$
- often performs better than that of using summated rating, even conditional independent and sufficient conditions do not hold.
- In general, a model based scaling method is always better than a simple addition approach based on intuition.

References

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  http://www.researchmethods.org/democracy-indicators.pdf