



Structural Equation Modeling

Mgmt 290
Lecture 3 – CFA and Hybrid Models
Oct. 10, 2005



Measurement is Everything

- Nothing can be done with wrong or unreliable measurements.
- “Measurement is Everything”.
- In research presentation or paper submission, measurement is the part being challenged the most.



Latent Variables are everywhere in research

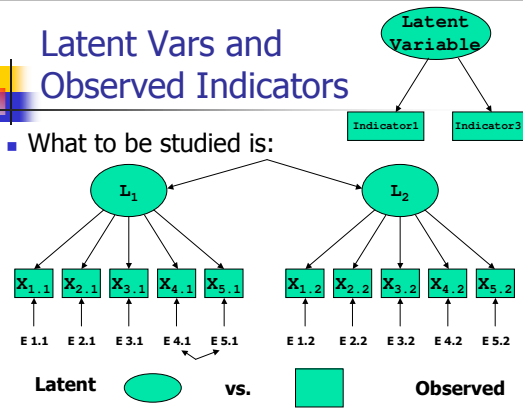
- “The true power of SEM comes from *latent variable modelling*”
- “Variables in psychology and other social science are rarely (never?) measured directly”
 - the effects of the variable are measured
 - Intelligence, self-esteem, depression
 - Reaction time, diagnostic skill
 - Democracy, Socio-Economical Status
 - Legitimacy, Management Skill
 - (soul, angels, ... - hypothetical construct)

Beyond Validity and Reliability: Between concept and indicators

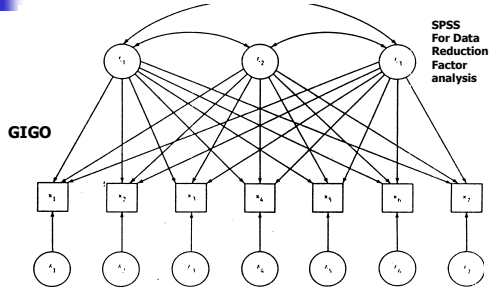
- Validity: Measures what it intends to measure.
 - Reliability: Consistency
 - Precision
 - repeatability

Latent Vars and Observed Indicators

- What to be studied is:



Exploratory Factor Analysis



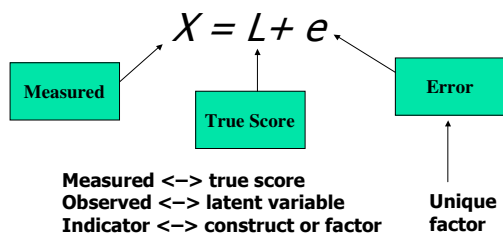
Confirmative Factor Analysis

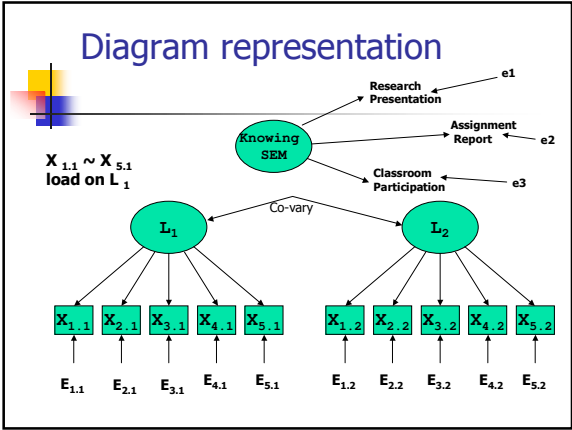
- 1) **E**quations & Diagrams: model representation
- 2) Identification & **E**stimation
- 3) **E**rrors and Evaluation: assumptions & fit indexes
- 4) **E**xplanation

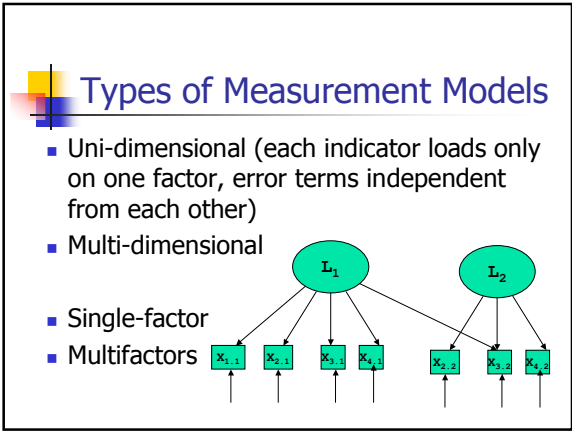
1) Equations & Diagrams: model representation

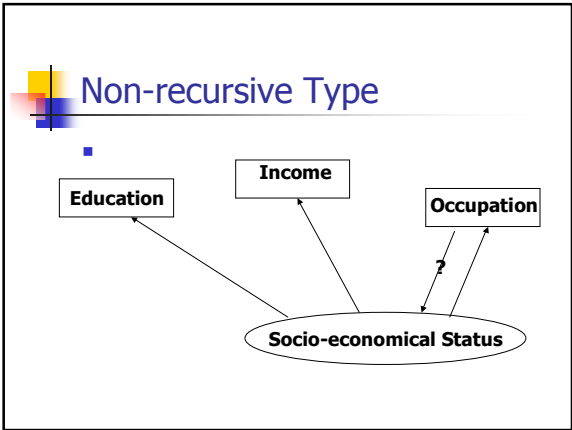
- $X_{1.1} = \varnothing_1 L_1 + e_{1.1}$
 - $X_{2.1} = \varnothing_2 L_1 + e_{2.1}$
 - $X_{3.1} = \varnothing_3 L_1 + e_{3.1}$
 - $X_{4.1} = \varnothing_4 L_1 + e_{4.1}$
- Loadings - $\varnothing_1 \dots$
 - $X \sim$ similar to endogenous variables
 - $L \sim$ similar to exogenous variables

More on Equations









2) Identification and Estimation

- Parameters \leq Observations
- Scale for every factor
- Single factor & ≥ 3 indicators
- 2 or more factors & 2 or more indicators per factor
- Less than 2 indicators for one or more factors
--- ???

Not an issue
As recursive

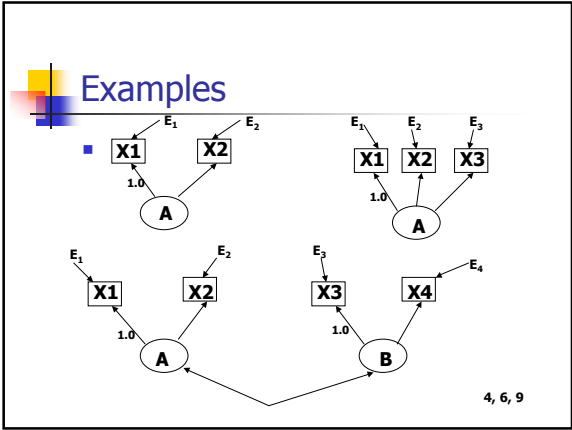
In literature, 3 indicators or 2 with 2 correlated factors or sample size > 200

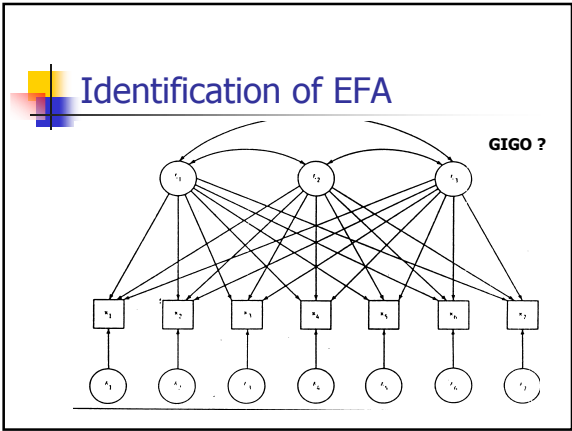
a) How to scale the latent variable

- 1) fix variances as a constant
- 2) fix one loading as 1

b) How to count

- # parameters = # loadings + vars & co-vas of factors + vars & co-vas of errors
- # obs = $v(v+1)/2$ ~ number of observed variables





- ### Estimation Methods
- ML – most often used
 - Generalized least squared
 - Un-weighted least squared

**3) Errors and Evaluation:
Assumptions**

- Multivariate normality

Fit Indices

- All the fit indices for path analysis applied to CFA
- Chi squared / df < 3
- GFI (Goodness Fit Index), AGFI close to 1
- SRMR (Standardized Root Mean Squared Residual) close to 0

**4) Explanation:
Factor loadings**

- Un-standardized coefficients
- (similar to regression coefficients)
- Standardized coefficients

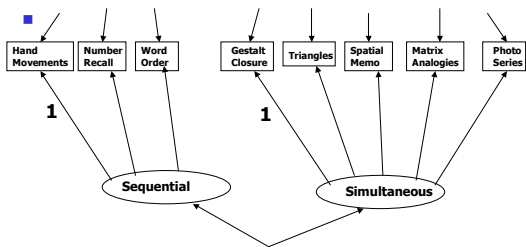


R²

- Proportion of explained variances
- (1 – measurement error variance / observed variances)
- 1-R² ~ proportion of unique variances



Example: The Model Representation

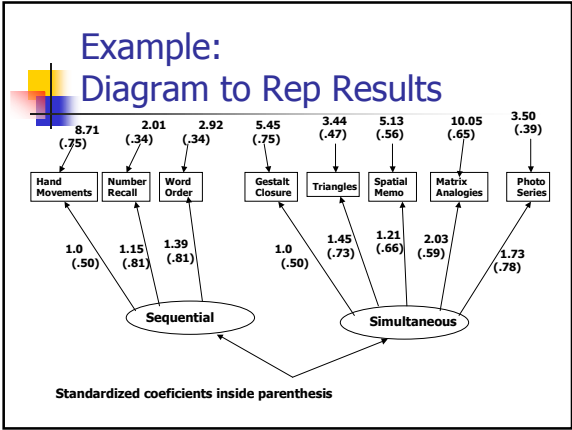


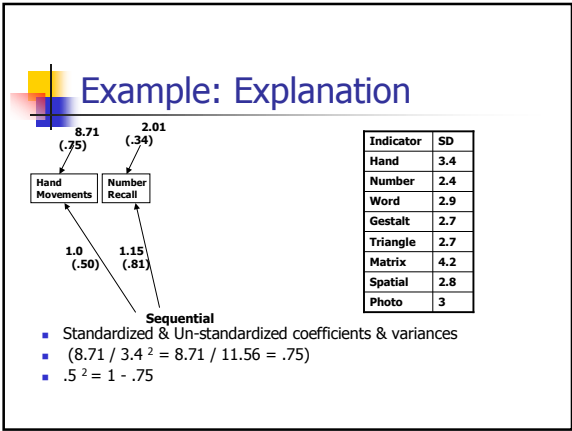


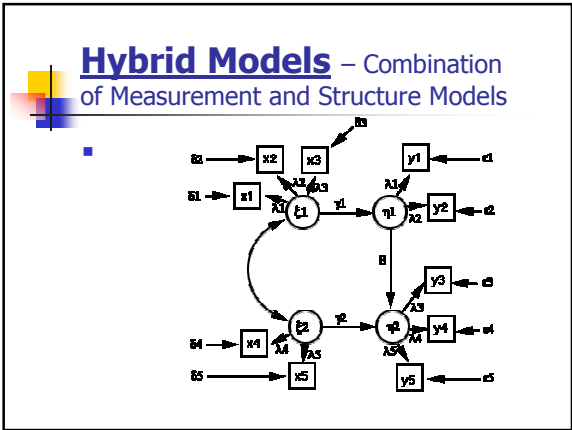
Example: Results

- R²
- Chi Square
- Chi-square = 38.13
- Df = 19 ~ 2-factor model
- For one factor
- 104.90 (df=20)

Indicator	R ²
Hand	.25
Number	.65
Word	.65
Gestalt	.25
Triangle	.52
Matrix	.43
Spatial	.35
Photo	.61





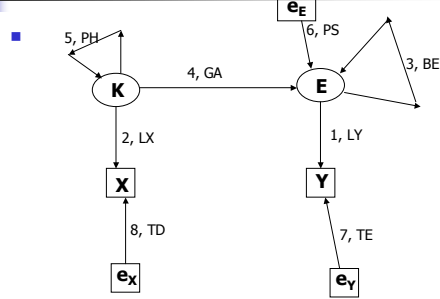


1) Equations and Diagram:

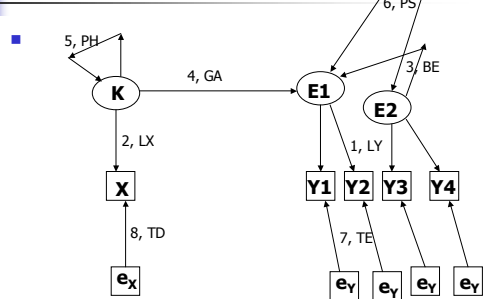
Model representation of Hybrid Model

- 6 Types of Terms
 - Observed Exogenous - X
 - Observed Endogenous - Y
 - Latent Exogenous - K
 - Latent Endogenous - E
 - Error Terms for Exogenous Obs - e_X
 - Error Terms for Endogenous Obs - e_Y

Diagram representation



More on Diagram representation



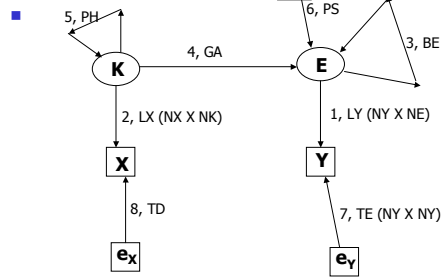


Model Representation

- NY = # observed endogenous
- NX = # observed exogenous
- NE = # latent endogenous
- NK = # latent exogenous



Model representation





2) Identification and Estimation

- Number of parameters $<(p+q)(p+q+1)/2$
- Two-Step Rule
 - Measurement Model Identification
 - Structural Model Identification



Estimation Methods

- ML again



3) Errors & Model Evaluation

- Fit Indexes

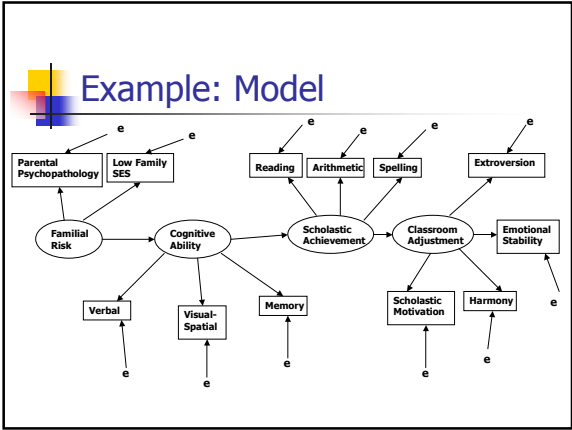
- Chi-squares

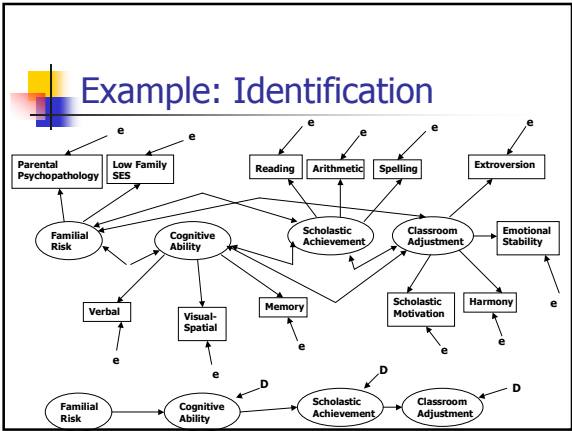


4) Explanation

- path coefficients

- and loadings





Example: Errors & Fix Indexes for Evaluation

- Better chi square/df for 3-factor measurement model (cognitive & scholar merger) (2.05 vs. 3.92)
- (also GFI and SRMR better)
- Good chi square/df for hybrid model
- (2.05)

Example: results explanation

